Biomedical and Mechanical Engineering Journal (BIOMEJ)
e-ISSN: 2776-1983, p-ISSN: 2829-5242
Vol. 3, No.1, May 2023, pp 25-29

The Application of Parabola of safety for Physics Problem Solving

Nailul Hasan[1]*

[1] Department of Physics, Faculty of Engineering, UPN Veteran Jatim
Jl. Rungkut Madya No.1, Gunung Anyar, Surabaya, Jawa Timur 60294

*Corresponding e-mail: nailul.hasan.fisika@upnjatim.ac.id

Abstract. In this article we calculate the parabola safety and its application for some physics problems. Using equation of motion for projectile we find the Parabola of safety. Deriving the parabola of safety here we used zero descriminant of quadratic equation. Inside parabola safety is unsafe region from projectile.The application of the equation is presented for finding maksimum distance, minimum speed in some cases of problems.

Keyword: Projectile motion; parabola of safety; physics problems

1. Introduction

Projectile motion is a combination of horizontal motion with constant velocity and constant acceleration vertical motion. The equation of horizontal motion is called uniform striated motion and vertical motion is called uniformly changing straight motion. The path equation resulting from the combination of the two is a parabolic equation. In this parabolic motion air resistance and the effect of the earth's rotation are ignored. In this article, we will derive the parabola of safety by using algebra approach. The equation is used to solve four example problems.

2. Methods

Let us start with basics equation of projectile position as function of time

$$x = v_0 \cos\theta t, \quad y = v_0 \sin\theta t - \frac{1}{2} g t^2 \quad (1)$$

Where v_0 is initial velocity, θ is angle intial velocity respect to the horizontal, t is time and g is gravitational acceleration.We take the initial position as center of coordinate.
By eliminating time we can get the trajectory equation

Biomedical and Mechanical Engineering Journal (BIOMEJ)
e-ISSN: 2776-1983, p-ISSN: 2829-5242
Vol. 3, No.1, May 2023, pp 25-29

$$y = x\tan\theta - \frac{1}{2v_0^2} gx^2\sec^2\theta \quad (2)$$

This is parabolic equation as function of position x

3. Result and Discussion

Using trigonometry identity $\sec^2\theta = 1 + \tan^2\theta$. We get quadratics equation in term of $\tan\theta$.

$$y = x\tan\theta - \frac{1}{2v_0^2} gx^2(1 + \tan^2\theta)$$

$$\frac{gx^2}{2v_0^2}(\tan^2\theta) - x\tan\theta + y + \frac{gx^2}{2v_0^2} = 0$$

By taking descriminant zero,

$$\tan\theta = \frac{x \pm \sqrt{x^2 - 4\frac{gx^2}{2v_0^2}\left(y + \frac{gx^2}{2v_0^2}\right)}}{2\left(\frac{gx^2}{2v_0^2}\right)}$$

We have condition for $\tan\theta$ is real value

$$x^2 - 4\frac{gx^2}{2v_0^2}\left(y + \frac{gx^2}{2v_0^2}\right) \geq 0$$

By little algebra we will arrive to the equation

$$y \leq \frac{v_0^2}{2g} - \frac{g}{2v_0^2}x^2 \qquad (3)$$

This equation tell us that for given horizontal position x the vertical position will be less than or equal as given by it.

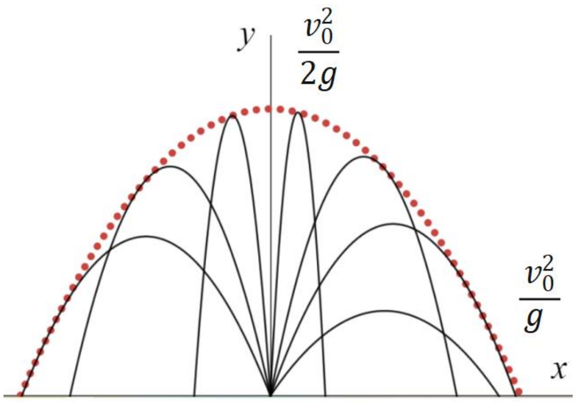

Figure 1. Envelop of parabola of safety

Biomedical and Mechanical Engineering Journal (BIOMEJ)
e-ISSN: 2776-1983, p-ISSN: 2829-5242
Vol. 3, No.1, May 2023, pp 25-29

Above equation is critical position which projectile can reach. Outside domain is safe from attack of projectile and inside parabola is unsafe region.

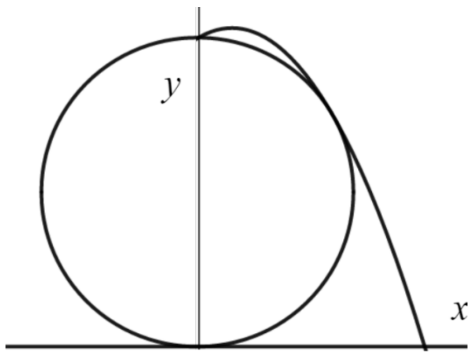

Figure 2. Optimum trajectory of projectile to reach the top of the sphere which is tangent to sphere and tangent to the envelop of parabola of safety

The first example for aplication is given of sphere of radius R, we want to find the minimal speed to reach the top from the ground. Let us assume the origin is the on the top sphere and projectile fired from the top having same path and speed if we fired projectile from the ground. In this problem we have circle equation .

$$x^2 + y^2 + 2yR = 0$$

Using the parabola of safety equation byy eliminating y.

$$x^2 + \left(\frac{v_0^2}{2g} - \frac{g}{2v_0^2}x^2\right)^2 + 2\left(\frac{v_0^2}{2g} - \frac{g}{2v_0^2}x^2\right)R = 0$$

The above equation can be simplified become biquadratic equation.

$$\left(\frac{g}{2v_0^2}\right)^2 x^4 + x^2\left(\frac{1}{2} - \frac{gR}{v_0^2}\right) + \frac{v_0^2}{g}\left(\frac{v_0^2}{4g} + R\right) = 0$$

By taking zero descriminant

$$\left(\frac{1}{2} - \frac{gR}{v_0^2}\right)^2 - 4\left(\frac{g}{2v_0^2}\right)^2 \frac{v_0^2}{g}\left(\frac{v_0^2}{4g} + R\right) = 0$$

Finally we get initial velocity of the top sphere

$$v_0 = \sqrt{\frac{gR}{2}}$$

The initial speed from the ground can be derived from conservation of energy

$$v = \sqrt{v_0^2 + 4gR} = 3\sqrt{\frac{gR}{2}}$$

Biomedical and Mechanical Engineering Journal (BIOMEJ)
e-ISSN: 2776-1983, p-ISSN: 2829-5242
Vol. 3, No.1, May 2023, pp 25-29

The second example is finding maksimum horizontal distance if projectile fired from a hill heigh h with intial speed v0. We can find maksimum distance by using

$$y = \frac{v_0^2}{2g} - \frac{g}{2v_0^2}x^2 = -h$$

With easy algebra we can find maksimum horizontal distance

$$x_{max} = \sqrt{\frac{2v_0^2}{g}\left(\frac{v_0^2}{2g} + h\right)}$$

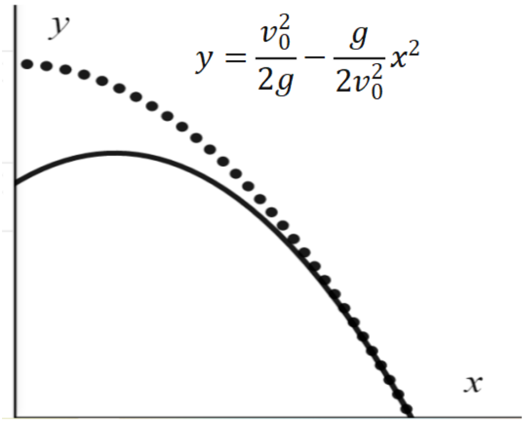

Figure 3. Optimum trajectory of projectile to reach maximum horizontal distance which is tangent to the envelop of parabola of safety

The third example is finding the minimum velocity to reach horizontal distance x=d and heigh y=h

$$\frac{v_0^2}{2g} - \frac{g}{2v_0^2}d^2 = h$$

We can rewrite in the form

$$\frac{v_0^4}{2} - v_0^2 gh - \frac{g^2 d^2}{2} = 0$$

Using abc formula for quadratic equation we find minimum initial speed is

$$v_0 = \sqrt{g\left(h + \sqrt{h^2 + d^2}\right)}$$

The last example of the application is finding maksimum distance in the case projectile motion fired in inclined plane. The angle the plane makes with the horizontal, φ. The projectile hits the incline plane at the position $x = L\cos\varphi$ and $y = L\sin\varphi$, where L is distance along the incline plane.

$$L\sin\varphi = \frac{v_0^2}{2g} - \frac{g}{2v_0^2}L^2\cos^2\varphi$$

By rearranging the equation we can the quadratic equation in term of L

$$\frac{g}{2v_0^2}L^2\cos^2\varphi + L\sin\varphi - \frac{v_0^2}{2g} = 0$$

Again using abc formula for quadratic equation we can easily get the distance

$$L = \frac{v_0^2}{g(1 + \sin\varphi)}$$

4. Conclussion

We have derived the equation of Parabola of Safety by taking zero descriminant in the quadratic equation in terms of $\tan\theta$. The region outside envelop is safe from prjectile. We have given some examples of its aplication for finding optimum distance or minimum initial velocity.

Acknowledgment

The author thanks to Dr. Intan Fatimah Hizbullah for his comments for the paper.

References

[1] Giancoli D C 2014 Physics: Principles with Applications 7th edn (Boston, MA: Pearson)
[2] Giambattista A, Richardson B M and Richardson R 2010 College Physics 3rd edn (New York: McGraw-Hill)
[3] Young H D and Freedman R A 2008 Sear's and Zemansky's University Physics: With Modern
[4] Physics 12th edn (Reading, MA: AddisonWesley) pp 79–87
[5] Tipler P A and Mosca G 2008 Physics for Scientists and Engineers 6th edn (New York: W. H. Freeman) pp 71–9
[6] Serway R A and Jewet J W 2004 Physics for Scientists and Engineers 6th edn (Belmont: Thomson Brooks/Cole) pp 83–91
[7] Palffy-Muhoray R and Balzarini D 1981 Maximizing the range of the shot put without calculus Am. J. Phys. 50 181
[8] Bose S K 1985 Maximizing the range of the shot put without calculus Am. J. Phys. 51 458
[9] Bace M et al 2002 Maximizing the range of a projectile Eur. J. Phys. 23 409
[10] Ganci S and Lagomarsino D 2014 Maximum range of a projectile launched from a height h: a non-calculus treatment Eur. J. Phys. 35 045026
[11] Bajc D 1990 Maximum range of a projectile motion—a generalization Am. J. Phys. 58 408
[12] Noll E D 1978 Simple projectile motion made simple Phys. Teach. 16 566
[13] Mohazzabi P and Kohken Z A 2005 Projectile motion without trigonometric functions Phys. Teach. 43 114
[14] Richard J-M 2004 Safe domain and elementary geometry Eur. J. Phys. 25 8